Exam. Code : 211003
Subject Code : 5501

M.Sc. (Mathematics) 3rd Semester

STATISTICS—I

Paper—MATH-577

Time Allowed—Three Hours] [Maximum Marks—100

Note :— Attempt **TEN** questions in all, selecting **TWO** questions from each unit. All questions carry equal marks.

UNIT—I

1. (a) What are the criteria of a good measure of the central tendency ? Also prove that the sum of the squared deviation is least when taken from the mean.

   (b) The average salary of male employees in a firm was Rs. 5,200 and that of females was Rs. 4,200. The mean salary of all the employees was Rs. 5,000. Find the percentage of male and female employees.

2. (a) What do you mean by skewness and kurtosis of a distribution ? Describe briefly their measures.

   (b) The standard deviation of symmetrical distribution is 5. What must be the value of the fourth central moment in order that the distribution be (i) Leptokurtic (ii) Mesokurtic (iii) Platykurtic ?

3. (a) Give the axiomatic definition of probability and describe the additive and subtractive property of its probability function.

(b) If a fair coin is tossed repeatedly, find the probability of getting m heads before obtaining n tails.

4. (a) Define conditional probability and state and prove Bayes Theorem of probability.

(b) A player tosses a coin and is to score one point for every head and two points for every tail that turned up. He is to play on until his score reaches or passes n. If p_n is the chance of attaining exactly n score, show that $p_n = \frac{1}{2}(p_{n-1} + p_{n-2})$ and hence find the value of p_n.

UNIT—II

5. (a) What is meant by random variable ? Also distinguish between discrete and continuous random variables and give the example of each type.

(b) A random process gives measurements x between 0 and 1 with probability density function :

$$f(x) = \begin{cases} 12x^3 - 21x^2 + 10x & , \quad 0 \le x \le 1 \\ 0 & , \quad \text{otherwise} \end{cases}$$

Find the value of (i) $P\left(x \le \frac{1}{2}\right)$ and $P\left(x > \frac{1}{2}\right)$

(ii) Constant k such that $P(x \le k) = \frac{1}{2}$.

6. (a) Define distribution function of random variable. Also describe a function which satisfies the properties of distribution function and explain.

(b) The joint probability density function of the two-dimensional random variable (x, y) is given by :

$$f(x, y) = \begin{cases} A(xy + e^x) & , \quad 0 < (x_1, x_2) < 1 \\ 0 & , \quad \text{otherwise} \end{cases}$$

Determine A and examine whether x and y are stochastically independent.

7. (a) What is meant by stochastic independence of random variables ? Also explain n-dimensional random variables.

(b) Let x_1, x_2 be a random sample of size 2 from a distribution with p.d.f.

$$f(x) = \begin{cases} e^{-x} & , \quad 0 < x < \infty \\ 0 & , \quad \text{otherwise} \end{cases}$$

Show that $y_1 = x_1 + x_2$ and $y_2 = \dfrac{x_1}{x_1 + x_2}$ are independent.

8. (a) Define marginal and conditional distributions for the case of three dimensional random variables.

(b) The random variables x and y have a joint p.d.f. f(x, y) given by :

$$f(x, y) = \begin{cases} \binom{y}{x} p^x (1-p)^{y-x} \dfrac{e^{-\lambda} \lambda^y}{y!} ; & x = 0, 1, 2, ... ; \\ & y = 0, 1, 2, ... ; \text{ with } y \geq x \\ 0 ; & \text{otherwise} \end{cases}$$

Find the marginal distribution of x and y. Also determine whether the random variables x and y are independent.

UNIT—III

9. (a) Define mathematical expectation of random variable and describe its properties.

(b) What is the expectation of the number of failures preceding the first success in an infinite series of independent trials with constant probability p of success in each trial ?

10. (a) State and prove Lindeberg-Levy central limit theorem.

(b) Let random variable x have the distribution :

$$P(x = 0) = P(x = 2) = p;\ P(x = 1) = 1 - 2p, \quad \text{for } 0 \leq p \leq \frac{1}{2}$$

For what p is the variance of x maximum ?

11. (a) State and prove Chebyshev's inequality.

(b) If x is a random variable such that $P_2(x) = 3$ and $P_2(x^2) = 13$, use Chebyshev's inequality to determine a lower bound for $P(-2 < x < 8)$.

12. (a) What is meant by moments ? Also define moment generating function and give its properties.

(b) Let $\{x_n\}$ be a sequence of independent Bernoulli variables such that $P(x_n = 1) = p_n = 1 - P(x_n = 0)$ for n = 1, 2, 3, ... Show that if

$$\sum_{n=1}^{\infty} p_n(1-p_n) = \infty$$, then the central limit theorem holds for the sequence $\{x_n\}$. What happens if

$$\sum_{n=1}^{\infty} p_n(1-p_n) < \infty.$$

UNIT—IV

13. (a) If random variable x have binomial distribution with parameters n and p, obtain the recurrence relation for its central moments.

(b) One worker can manufacture 120 articles during a shift, another worker 140 articles, the probabilities of the articles being of a high quality are 0.94 and 0.80 respectively. Determine the most probable number of high quality articles manufactured by each worker.

14. (a) What is hypergeometric distribution ? Find its mean and variance.

(b) For a geometric distribution with p.m.f. :

$$f(x) = 2^{-x} \; ; \; x = 1, 2, 3, ...$$

Show that Chebyshev's inequality gives $P\{|x - 2| \leq 2\} > \frac{1}{2}$, while the actual probability is $\frac{15}{16}$.

15. (a) State and prove that geometric distribution lacks memory.

(b) If random variable has a uniform distribution in [0, 1], find the distribution of $y = -2 \log x$. Identify the distribution also.

16. (a) Define the Beta distribution of first and second kind. Also find mean and variance of its first kind.

(b) The random variables x and y are independent, each exponentially distributed with same parameter θ. Find the distribution of $\frac{x}{x+y}$ and identify its distribution.

UNIT—V

17. (a) What do you mean by correlation between the random variables ? How is it measured between two random variables ? How can you use scatter diagram to obtain an idea of the correlation ?

(b) Let $U = ax + by$ and $V = ax - by$, where x, y represent deviations from the means of two measurements on the same individual. If U, V are uncorrelated, show that :

$$\sigma_u \sigma_v = 2\, ab\, \sigma_y \sigma_x (1 - \rho^2)^{1/2}.$$

Here $\sigma_u^2, \sigma_v^2, \sigma_x^2, \sigma_y^2$ and ρ denote the respective variances and correlation coefficient.

18. (a) Explain partial correlation and multiple correlation. In the usual notations, prove that

$$R_{1.23}^2 = \frac{r_{12}^2 + r_{13}^2 - 2r_{12}r_{23}r_{31}}{1 - r_{23}^2} \leq r_{12}^2 .$$

(b) The job rating efficiency of an employee seems to be related to the number of weeks of employment. For a random sample of 10 employees, the following data were observed :

Job efficiency (x)	**Weeks of Employment (y)**
55	2
50	4
20	1
55	3
75	5
80	9
90	12
30	2
75	7
70	5

Obtain a regression line of x on y.

19. (a) When are two attributes said to be (i) positively associated and (ii) Negatively associated ? Also define complete association and dissociate of two attributes.

(b) Given that $X = 4Y + 5$ and $Y = kX + 4$ are the lines of regression of X on Y and Y on X respectively, show that $0 < 4k < 1$. If $k = \frac{1}{16}$, find the means of the two variables and coefficient of correlation between them.

20. (a) Define correlation ratio η_{xy} and prove that $1 \geq \eta^2_{xy} \geq r^2$, where r is the correlation coefficient between x and y.

(b) Obtain regression equation of line of Y on X for the following distribution :

$$f(x, y) = \frac{y}{(1+x)^4} e^{-\frac{y}{1+x}} \; ; \; x, y \geq 0.$$